Stress Analysis of the Sarafix External Fixator Design

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Abstract: A stress analysis of the Sarafix external fixator design was performed using finite element analysis (FEA) and experimental tensometric measurements. The study was conducted at one of the Sarafix fixator configurations that have a clinical application in the treatment of tibia fractures. The intensity of principal and von Mises stresses generated at two measuring points (MP) on the fixator connecting rod were monitored and analyzed during the testing on axial compression on the fixator design and its finite element model (FEM). The 3D geometrical and FEM model of the fixator was formed using the computer aided design/computer aided engineering (CAD/CAE) software system CATIA. Verification of the results for the dominant principal stresses obtained from FEA was carried out through tensometric measurements. The measuring chain consisted of strain gauges connected into two Wheatstone half-bridges, digital measuring amplifier system and a computer with software for acquisition and monitoring of measurement results. A quite good agreement was observed between the results obtained on the basis of FEA and results of experimental tensometric analysis.

Key words: Finite element analysis, tensometric measurements, principal stresses, von Mises stress, Sarafix external fixator.

1. Introduction

After J.F. Malgaigne invented the external fixator in 1840, their selection and application was generally carried out on empirical grounds and accumulated experience in clinical orthopedics and traumatology. In order to promote and carry out necessary research to improve fixation, a development of a theoretical analysis of problems fixation based on the principles of structural mechanics is pursued. With the aim of determining mechanical characteristics of external fixators, various sensors and transducers are set up on their designs [1]. During the past few years, except for performing the experimental testing, there has been an increased use of 3D modeling and FEA, in order to more fully describe the behavior of the fixator and its components during the loading [2].

The external fixator is a medical device for the immobilization of fractures or serious damage to the structure of extremities. External fixation is a method of fracture immobilization achieved by the application of pins or wires into or through a bone and their binding to the outer frame. The above basic concept of the method has not changed since its origin, but progress is reflected through the development of new design solutions and materials used.

The most complicated aspect of bone fractures, both in terms of complexity of treatment and structural stresses of external fixator, is an open fracture. In the case of open fractures, in the initial phase of treatment, the full load is transferred through the fixator.

This paper presents the results of a stress analysis of the unilateral uniplanar configuration of the Sarafix external fixator (Fig. 1) in the case of load under axial compression. Otherwise, the most significant load of the external fixator during the postoperative treatment of patients is the axial compression itself.

An open fracture at the middle of tibia with the fracture gap of 50 mm was examined. The analyzed configuration of the Sarafix fixator contains four half-pins in the proximal and distal segment of a bone placed in one plane.
The paper is organized as follows: Section 2 presents used methods of work; section 3 is finite element analysis of the fixator design; section 4 introduces experimental tensometric analysis; section 5 presents results; section 6 gives conclusions and future work.

2. Objective and Methods

Complete mechanical research of the fixator, besides the examination of its rigidity to the loads to which it was exposed after the application, includes the analysis of stresses (von Mises and principal stresses) on the characteristic location of fixator designs. Mechanical testing of Sarafix fixator was not performed before its clinical application, because of the war-time circumstances in which it originated. Extensive studies of the mechanical research of the Sarafix fixator were carried out within the master’s thesis [3].

Geometrical modeling of the Sarafix fixator and FEA were carried out at the Laboratory for Computer aided design—CADlab of the Faculty of Mechanical Engineering Sarajevo. The first step consisted of forming a 3D geometrical model of the analyzed Sarafix fixator configuration, whereupon the FEA was performed on the model using CAD/CAE software system CATIA. During the structural FEA, values of von Mises stresses were observed at two control points in the middle of the fixator connecting rod. The intensity and direction of principal stresses were monitored and analyzed at the same points.

Experimental testing was conducted at the Laboratory for materials testing and Laboratory for machine elements. At the Laboratory for materials testing, the examination of the analyzed configuration of the Sarafix fixator on the axial compression was performed, using a universal material testing machine (Zwick GmbH & Co., Ulm, Germany, model 143501). During the testing, the intensity of the load on the model of proximal segment of the tibia was controlled, using the force transducer. A wooden model of the proximal and distal bone segments are supported on the ball joint supports.

Tensometric measurement equipment (Laboratory for machine elements) was used to control and monitor the value of the dominant principal stress on the two measurement points at the middle of the fixator connecting rod. The following equipment from the HBM (Hottinger Baldwin Messtechnik GmbH, Darmstadt, Germany) manufacturer was used:

- Digital measuring amplifier system—digitales messverstarker-system (DMC) 9012A;
- Computer with software for acquisition, monitoring and processing of measurement results—Catman; and
- Four strain gauges (type 3/120LY11) connected in two Wheatstone half-bridges.

The strain gauges were placed on the opposite sides of the Sarafix fixator connecting rod at the same locations where intensities of maximum and minimum principal stresses were monitored during the FEA. Thereafter, the strain gauges were connected with the DMC system and computer through two separate channels. In this way, the maximum and minimum principal strains on the measuring points were measured independently. This measurement method was applied because the connecting rod was subjected to a compound strain, which consisted of bending strain and axial compressive strain.
3. FEA of the Fixator Design

Understanding the physical behavior of the model is a basic prerequisite for successful process of modeling real systems. Before that, it is necessary to make numerous assumptions related to modeling: structure, joints between the components, boundary conditions, loads, materials, etc.

During the processes of the linear FEA, the material of wooden bone models was defined as orthotropic, while materials of the fixator design were modeled as isotropic. The FEM model consisted of solid finite elements of a linear (TE4) and parabolic tetrahedron (TE10) type (Fig. 2).

Join elements of the spider type were used for modeling the joints between the components of the Sarafix fixator [3]. The following joints were used: Fastened connection, Contact connection and Bolt tightening connection. The modeling of the influence of supports was performed using a Smooth virtual part. Fig. 3 shows the CAD and FEM model of the analyzed Sarafix fixator configuration after preprocessing.

At the end of the proximal bone segment, the axial load in the form of surface force (Force density) was applied in the direction of the z axis of the Cartesian coordinate system. A displacement constraint of the Sarafix FEM model was derived by using the Ball join restraint on the model of distal bone segment. Likewise, a displacement constraint at the model of proximal bone segment was performed by using the User-defined restraint, which prevented the two translations in direction of x and y axis of the Cartesian coordinate system (Fig. 3).

The principal stresses of the stress tensor are the distinctive values of the stress tensor, while their direction vectors are the principal directions or eigenvectors [4]. When the coordinate system is chosen to coincide with the eigenvectors of the stress tensor, the stress tensor is represented by a diagonal matrix:

\[
\sigma = \begin{bmatrix}
\sigma_1 & 0 & 0 \\
0 & \sigma_2 & 0 \\
0 & 0 & \sigma_3
\end{bmatrix}
\]  

(1)

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses.

The values of the principal and von Mises stress were controlled on two locations at the middle of the fixator connecting rod during the FEA. The measuring point closer to the model of the bone segment was marked with MP- and the point on the opposite side of the connecting rod was marked with MP+ (Fig. 4).

Compressive stresses, which were recorded at the measuring point MP- have a higher intensity compared to the tensile stress at the MP+. This is a direct consequence of the appearance of an eccentric compression that exposed fixator connecting rod.
The direction of the maximum principal stress ($\sigma_1$) on the measuring point MP+ coincides with the direction of $z$ axis, i.e., the axis of symmetry of the connecting rod. Likewise, the direction of the minimum principal stress ($\sigma_3$) on the MP- coincides with the axis of symmetry of the connecting rod. The minimum principal stress compared to the other two principal stresses at the MP- is dominant. Within Fig. 4 a view B is given where directions and intensities of the principal stresses on the measuring points are presented. Note that at the MP+ the maximum principal stress is in fact the tensile stress, while at the MP- the minimum principal stress is actually the compressive stress.

Also, it can be seen that the dominant principal stresses ($\sigma_1$ and $\sigma_3$) are in the bending plane of the fixator which is not parallel with the plane of the one-half pins. For this reason, the vectors of the dominant principal stresses do not match either (Fig. 4, View B).

A quantity called the equivalent stress or von Mises stress is commonly used in solid mechanics to predict yielding of materials under multiaxial loading conditions using the results from simple uniaxial tensile tests. The equivalent stress is defined as

$$\sigma_e = \sigma_{eq} = \sqrt{\frac{1}{2} J_2} = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$  \hspace{1cm} (2)

where $J_2$ is the second deviatoric stress invariant.

The von Mises stress is equivalent to the maximum distortion strain energy and it is a good indicator of the yielding of materials. By analyzing the distribution of von Mises stress fields shown in Fig. 5, it can be concluded that the highest stresses on the fixator design did not occur at the measuring point.

The maximum value of von Mises stress at the measuring points was $\sigma_{eq} = 355$ MPa. Generally, the highest von Mises stress on the Sarafix fixator design occurred in the contacts between the connecting rod and the clamping ring ($\sigma_{eq} = 550$ MPa) (Fig. 5).

The element stresses at Gauss points may be expressed as

$$\sigma = D\varepsilon$$  \hspace{1cm} (3)
4. Experimental Tensometric Analysis

The analyzed configuration of the Sarafix fixator was attached to proximal and distal tibia bone segments modeled with cylindrical wooden bars with known physical properties. Intensities of the loads were determined based on in-vivo testing on patients [1]. During the axial compression testing (Fig. 6), the tibia bone models were supported on ball joints and, using the force transducer, the axial load was controlled in the range of 0 to 600 N at the rate of 5 N/s.

The connecting rod, due to the axial compression at the proximal segment of the bone model, is exposed to the combined loading (eccentric pressure), which consists of a combination of bending and axial compression. This form of the strain is manifested by the unequal distribution of tensile and compression stresses along the longitudinal section of a connecting rod, i.e., neutral line does not coincide with the axis of symmetry of the fixator connecting rod. Therefore, the two separate Wheatstone half-bridges were formed and connected with the DMC system via two measurement channels. Wheatstone half-bridges consist of active strain gauge SG1 and compensation (inactive) strain gauge SG2 (Figs. 6-7). The compensating strain gauges were placed near the active strain gauges on a plate tied to a connecting rod (Fig. 6). The plate and connecting rod are made of the same material.

The strain, registered by Wheatstone half-bridge with one active and one compensation strain gauge, is given by the relation [5]:

$$\varepsilon = \frac{4}{k} \frac{U_A}{U_E}$$

where:
- $k$—gauge factor;
- $U_A$—bridge output voltage;
- $U_E$—excitation voltage (bridge input).

Compensating strain gauges are used to compensate the effect of temperature on the measurement and they are of the same type as the active ones. In this way, it is possible to determine the intensity of the dominant principal stresses at the measuring points. Previously
performed FEA determined the direction and intensity of the principal stresses.

Also, it was noted that the intensities of the other two principal stresses at the measuring points were negligible compared to the maximum ($\sigma_1$ on MP+) and minimum ($\sigma_3$ on MP-) principal stress (Table 1).

Active strain gauges are placed on the opposite sides of the connecting rod at the nearest and farthest point from the model of the bone, so that their longitudinal axis coincides with the directions of dominant principal strains ($\varepsilon_1$ and $\varepsilon_3$) at the measuring points.

Simultaneous measuring of the largest positive and negative principal strains on the opposite sides of the fixator connecting rod was carried out independently at two measurement points (Fig. 8). In the following analysis, the strain gauge placed on the side of the connecting rod closer to the bone model will be referred to as SG-, while a strain gauge placed on the opposite side will have a label SG+.

This way of setting up strain gauges enables the measurement of the greatest positive principal strain ($\varepsilon_1$) at the measuring point MP+, on the basis of which the intensity of the maximum principal stress ($\sigma_1$) is determined. Analogously, on the measuring point MP-, the greatest negative principal strain ($\varepsilon_3$) was measured, on the basis of which the intensity of the minimum principal stress ($\sigma_3$) is determined. The minimum principal stress compared to the other two principal stresses at the point MP- is dominant.

Independently measured total strains at the measuring point consisted of the compressive and bending strain [6-8]. The total (principal) strains are defined by the principle of superposition, as follows:

$$\varepsilon_1 = -\varepsilon_p + \varepsilon_s = -\frac{F}{AE} + \frac{M}{EZ}$$

$$\varepsilon_3 = -\varepsilon_p - \varepsilon_s = -\frac{F}{AE} - \frac{M}{EZ}$$  (7)

where:

$\varepsilon_p$—the strain component caused by the axial compressive force;

$\varepsilon_s$—the strain component caused by the bending moment;
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Fig. 8  Arrangement of strain gauges and distribution of loads and strains in the longitudinal section of the connecting rod.

- $F$—the axial compressive force;
- $A$—the area cross-section of the fixator connecting rod;
- $E$—modulus of elasticity;
- $M$—bending moment;
- $Z$—section modulus of the fixator connecting rod.

In this case of load, the bending strain was significantly higher than the compression strain ($|\varepsilon_3| >> \varepsilon_1$) Distribution of the strains in the longitudinal section of the fixator connecting rod is shown schematically in Fig. 8.

The dominant principal stresses at the measuring points (MP+ and MP-) are determined through the relations:

$$
\sigma_1 = \varepsilon_1 E \\
\sigma_3 = \varepsilon_3 E
$$

(8)

Acquisition, display and processing of measurement results are performed using the HBM Catman software.

5. Results

In order to achieve a direct comparison of results of the FEA and tensometric analysis, all parameters of geometry, materials, loads, restraints on the FEM model are set according to experimental settings.

Tables 1-2 show the intensities of main and von Mises stresses generated at the measuring points in the case of maximum axial compression force. The value of the maximum principal stress ($\sigma_1$) at the MP+ was significantly higher than the other two principal stresses ($\sigma_2$ and $\sigma_3$). On this basis, and bearing in mind the relationship by which the value of von Mises stress are calculated (relation 2), it follows that at the MP+ the von Mises stress ($\sigma_{vm}$) has the same value as the maximum principal stress ($\sigma_1$).

Likewise, the value of the minimum principal stress ($\sigma_3$) at the MP- was significantly higher than the other two principal stresses ($\sigma_1$ and $\sigma_2$). Analogously as in the previous case, it follows that the von Mises stress ($\sigma_{vm}$) is equal to the minimum principal stress ($\sigma_3$) at the MP- (Table 2).

The maximum deviations of the results obtained by FEA in relation to the results obtained by experimental testing are range: the principal stress $\sigma_1$ to 1.2%, and

<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma_1$ (MPa)</th>
<th>$\sigma_2$ (MPa)</th>
<th>$\sigma_3$ (MPa)</th>
<th>$\sigma_{vm}$ (MPa)</th>
</tr>
</thead>
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<tr>
<td>FEA</td>
<td>330</td>
<td>0.2</td>
<td>0.001</td>
<td>330</td>
</tr>
<tr>
<td>Experiment</td>
<td>334</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma_1$ (MPa)</th>
<th>$\sigma_2$ (MPa)</th>
<th>$\sigma_3$ (MPa)</th>
<th>$\sigma_{vm}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA</td>
<td>-0.003</td>
<td>-0.4</td>
<td>-355</td>
<td>355</td>
</tr>
<tr>
<td>Experiment</td>
<td>-</td>
<td>-</td>
<td>-368</td>
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the principal stress $\sigma_3$ to 3.6% (Fig. 9). Principal stresses with the negative sign represent compressive stress. It is noted that at the MP+ all principal stresses are with positive signs (Table 1), while at the MP- all principal stresses are with negative signs (Table 2). The maximum values of von Mises and maximum principal stress at the control points is respectively $\sigma_{vm} = 355$ MPa and $\sigma_3 = 368$ MPa and they are lower than the yield strength of the material of the fixator connecting rod ($\sigma_Y = 650$ MPa).

6. Conclusions

The conducted research has shown that there is a linear dependence between the loads and stresses generated on the fixator connecting rod, as a result of the absence of large displacement and plastic deformation of the fixator components.

Comparing the results of FEA and tensometric analysis of the principal stresses at the measuring points reveals their good agreement and argues that the solutions obtained by FEA were verified.

The CATIA software system can be successfully used in the development of CAD models, FEA and computer simulations of the process from different areas of technics and medical engineering. Using the developed CAD/FEM model of the Sarafix fixator, it is possible to control displacement and stresses generated at any point of the bone-fixator system and then make possible corrections on the fixator design. Due to extreme flexibility of the formed 3D geometrical model, rapid changes were enabled not only to the geometry and position of components and fixator, but also to the materials applied in the external fixation (medical stainless steel, composite materials, titanium alloys).

In this way, conditions for design optimization of the external fixator are created, which would significantly shorten time and reduce costs of development of medical devices for external fixation of bones. In addition, the application of such models greatly reduces the volume of conventional preclinical experimental testing of fixators.

References