Position Sensorless Torque Estimation of Synchronous Reluctance Motors Based on Extended-Flux Model without Core-Loss Measurement

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Abstract: This paper proposes an extended-flux model with core-loss resistance of SynRMs (synchronous reluctance motors) and precise torque estimation without core-loss measurement and position encoder. The proposed torque estimation is useful for precise MTPA (maximum torque per ampere) control of position sensorless controlled SynRMs, which is achieved with the assistance of active and reactive powers.

Key words: Synchronous reluctance motors, maximum torque per ampere control, torque estimation, magnetic saturation, position sensorless.

1. Introduction

This paper proposes an extended-flux model with core-loss resistance of SynRMs (synchronous reluctance motors) and precise torque estimation without core-loss measurement and position encoder, which enables us to achieve precise MTPA (maximum torque per ampere) control of SynRMs.

SynRMs are known to possess magnetic non-linearity and considerable core-loss, which make it difficult to realize MTPA control and maximum efficiency control.

If synchronous reluctance motors did not possess magnetic non-linearity and core-loss, the control phase in current vector (“the current control phase”, hereafter) could be fixed to 45° for MTPA control and high efficient drives. In reality, however, the current control phase should be assigned to more than 45° because of magnetic saturation in d-axis, and this phase should appropriately be advanced furthermore for high efficient drive and less core-loss [1].

Many researchers have already tackled this problem, that is appropriate design of the current control phase, so far [2-6]. All of these interesting approaches are based on the mathematical model, which depends on precise measurements of the motor parameters such as d-q stator inductances or stator flux, and equivalent core-loss resistance. In general, these parameters should be measured with complicated procedures in advance, yielding much time-consuming.

On the other hand, some attractive position sensorless vector controls of SynRMs have already proposed [7, 8] for low-cost high performance drive. It should be noted that Ichikawa et al. [8] have already established parameter identification in position sensorless control to solve magnetic saturation characteristics. These approaches, however, also depend strongly on the SynRM mathematical model with magnetic characteristics. This implies that these approaches would inherently hold same problems as...
above torque control and high efficient drives.

This paper proposes precise torque estimation of synchronous reluctance motors without core-loss measurement and position encoder based on an extended-flux model with equivalent core-loss resistance. First of all, this paper obtains a new extended-flux model for SynRMs in consideration of core-loss, which enables us to solve some problems caused by magnetic saturation effect. Estimation of the developed torque of SynRM and the equivalent core-loss resistance is then proposed with the assistance of active and reactive powers, which can realize precise developed torque estimation with only the resistance and the q-axis inductance, whose values are relatively kept constant. Finally, this paper demonstrates the feasibility of the proposed strategy by some experimental results.

2. Extended Flux Model of SynRMs with Equivalent Core-Loss Resistance

2.1 Definition of Extended Flux Model of SynRMs

In general, the electrically developed torque of SynRM \( \tau_e \) can be expressed by:

\[
\tau_e = (L_{1d} - L_{1q})i_{1d}i_{1q} = |\lambda|i_{1q} \tag{1}
\]

where \( L_{1d} \) and \( L_{1q} \) are the winding inductances of \( d-q \) axes, respectively, and \( i_{1d} \) and \( i_{1q} \) represent the winding currents on \( d-q \) axes, respectively. \( |\lambda| \) is defined as:

\[
|\lambda| = L_{1d} - L_{1q}i_{1d} \tag{2}
\]

Eq. (1) implies that this flux aligns with \( d \)-axis. In this paper, this flux \( \lambda \) is named the extended flux.

2.2 Extended Flux Model of SynRMs without Equivalent Core-Loss Resistance

Next, this subsection briefly reviews the extended flux model of SynRMs [9]. The model in this paper can be expressed on the stationary coordinate (\( \alpha-\beta \)) and without \( L_d \). Therefore, this flux model is insensitive to \( d \)-axis flux saturation and cross-coupling due to \( i_q \). As a result, this makes it possible robust flux estimation to \( d \)-axis magnetic saturation. Let \( v_{1d}, v_{1q}, i_{1d}, \text{ and } i_{1q} \) be the voltages and currents on the \( d-q \) axes, respectively. In general, a mathematical model of SynRM is expressed on the \( d-q \) axes as:

\[
\begin{bmatrix}
  v_{1d} \\
  v_{1q}
\end{bmatrix} =
\begin{bmatrix}
  R_1 + pL_{1q} & 0 \\
  -\omega_mL_{1d} & R_1 + pL_{1q}
\end{bmatrix}
\begin{bmatrix}
  i_{1d} \\
  i_{1q}
\end{bmatrix} \tag{3}
\]

where,

\( v_{1d}, v_{1q} \): winding voltages in \( d-q \) axes;
\( i_{1d}, i_{1q} \): winding currents in \( d-q \) axes;
\( R_1 \): winding resistance;
\( \omega_m \): rotor speed in electrical angle;
\( p \): differential operator.

The above equation can be manipulated as follows:

\[
\begin{bmatrix}
  v_{1d} \\
  v_{1q}
\end{bmatrix} =
\begin{bmatrix}
  R_1 + pL_{1q} & 0 \\
  -\omega_mL_{1d} & R_1 + pL_{1q}
\end{bmatrix}
\begin{bmatrix}
  i_{1d} \\
  \frac{(L_{1d} - L_{1q})i_{1d}}{\omega_m}
\end{bmatrix} + \begin{bmatrix}
  0 \\
  \omega_mL_{1d}
\end{bmatrix}i_{1q} \tag{4}
\]

In addition, transforming this equation into the \( \alpha-\beta \) coordinates, the following equation can be obtained:

\[
\begin{bmatrix}
  v_{\alpha} \\
  v_{\beta}
\end{bmatrix} =
\begin{bmatrix}
  R_1 + pL_{1q} & 0 \\
  -\omega_mL_{1d} & R_1 + pL_{1q}
\end{bmatrix}
\begin{bmatrix}
  i_{\alpha} \\
  i_{\beta}
\end{bmatrix} + \begin{bmatrix}
  0 \\
  \omega_mL_{1d}
\end{bmatrix}i_{\beta} \tag{5}
\]

Substituting the following flux definition into Eq. (5), Eq. (6) can be obtained.

\[
\begin{bmatrix}
  v_{\alpha} \\
  v_{\beta}
\end{bmatrix} =
\begin{bmatrix}
  R_1 + pL_{1q} & 0 \\
  -\omega_mL_{1d} & R_1 + pL_{1q}
\end{bmatrix}
\begin{bmatrix}
  i_{\alpha} \\
  i_{\beta}
\end{bmatrix} + \begin{bmatrix}
  0 \\
  \omega_mL_{1d}
\end{bmatrix}i_{\beta} + \begin{bmatrix}
  \lambda_{\alpha} \\
  \lambda_{\beta}
\end{bmatrix} \tag{6}
\]

It should be noted that only \( L_{1q} \) appears in the resulting mathematical model as the inductance parameter, and this formulation is equal to that of SPMSM. This implies that SynRM with large saliency can be considered to be the same as if SPMSM without any approximation. In other words, this model formulation results in robustness to \( L_d \) variation due to magnetic saturation, and flux \( \lambda \) can be estimated based on this model that is insensitive to \( L_d \).
2.3 Extended Flux Model of SynRMss Taking Equivalent Core-Loss Resistance into Consideration

This subsection describes an extended flux model of SynRMs with equivalent core-loss resistance, which is originated from the equivalent eddy-current circuit [10], which would be the basis on accurate torque estimation for MTPA control. The SynRM model with the equivalent eddy-current circuit [11] is depicted in Fig. 1, and is given in steady state by:

\[
\begin{bmatrix}
    \frac{v_{sd}}{v_{sq}} \\
    \frac{v_{qd}}{v_{qg}} \\
    0
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

in which:
- \(i_{sd}, i_{sq}\): currents of equivalent core-loss circuit in \(d-q\) axes;
- \(R_{i}\): resistance of equivalent core-loss circuit;
- \(L_{sd}, L_{sq}\): inductances of equivalent core-loss circuit in \(d-q\) axes;
- \(M_{d}, M_{q}\): mutual inductances in \(d-q\) axes.

This equation can be rewritten by:

\[
\begin{bmatrix}
    \frac{v_{sd}}{v_{sq}} \\
    \frac{v_{qd}}{v_{qg}} \\
    0
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

Eliminating the magnetic saliency of SynRMs the same manner as previous, this paper employs the following extended-flux:

\[
\lambda_{ldd} \equiv (L_{1d} - L_{1q})i_{ld} + (M_{d} - M_{q})i_{ld}
\]  

(8)

Assuming the leakage inductances between the stator windings and the equivalent eddy-current circuits are neglectable, \(L_{1d} \approx L_{1d} \approx M_{d}, L_{1q} \approx L_{1q} \approx M_{q}\) are hold, so that the above voltage equation can be expressed by:

\[
\begin{bmatrix}
    \frac{v_{sd}}{v_{sq}} \\
    \frac{v_{qd}}{v_{qg}} \\
    0
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

(9)

Next, Eq. (9) is simplified. The currents in the equivalent core-loss circuit \(i_{sd}, i_{sq}\) can be obtained from the third and fourth rows in Eq. (9):

\[
\begin{align*}
\frac{v_{sd}}{v_{sq}} \\
\frac{v_{qd}}{v_{qg}} \\
0
\end{align*}
\]

\[
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

According to Ref. [11], \(R_{3} \approx \frac{w^{2}rL_{3}M_{q}}{w^{2}rL_{3}M_{q}}\) is assumed in this paper. This assumption implies that the time-constant in the equivalent eddy-current circuit is small enough. Hence, Eq. (10) is simplified into:

\[
\begin{align*}
\frac{v_{sd}}{v_{sq}} \\
\frac{v_{qd}}{v_{qg}} \\
0
\end{align*}
\]

\[
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

(11)

On the other hand, the first and second rows in Eq. (7) can be manipulated as:

\[
\begin{align*}
\frac{v_{sd}}{v_{sq}} \\
\frac{v_{qd}}{v_{qg}} \\
0
\end{align*}
\]

\[
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

(12)

Substituting Eq. (11) into Eq. (12),

\[
\begin{align*}
\frac{v_{sd}}{v_{sq}} \\
\frac{v_{qd}}{v_{qg}} \\
0
\end{align*}
\]

\[
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

(13)

can be obtained. Assuming \(R_{3} \approx \frac{w^{2}rL_{3}M_{q}}{w^{2}rL_{3}M_{q}}\) again, and defining \(R_{m} = (\omega_{c}M_{q})^{2}/R^{3}\) as the equivalent core-loss resistance, the extended-flux model for SynRM with equivalent core-loss circuit can be obtained as the following:

\[
\begin{align*}
\frac{v_{sd}}{v_{sq}} \\
\frac{v_{qd}}{v_{qg}} \\
0
\end{align*}
\]

\[
\begin{bmatrix}
    i_{sd} \\
    i_{sq} \\
    i_{qd} \\
    i_{qg}
\end{bmatrix} =
\begin{bmatrix}
    \frac{R_{i}}{\alpha_{c}\omega_{L}} & -\frac{\alpha_{c}\omega_{L}M_{d}}{\omega_{L}} & 0 & -\frac{\alpha_{c}\omega_{L}M_{q}}{\omega_{L}} \\
    0 & \frac{R_{i}}{\omega_{L}} & \frac{R_{i}}{\omega_{L}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    v_{sd} \\
    v_{sq} \\
    v_{qd} \\
    v_{qg}
\end{bmatrix}
\]

(14)
It should be noted that:

(1) Eq. (14) is constructed without $L_d$, which extremely varies due to magnetic saturation effect. Hence, the characteristic of this model is insensitive to magnetic non-linearity in $d$-axis;

(2) $R_m$ is incorporated as the coefficients for $\lambda_{1d}$ as well as the currents $i_d$ and $i_q$. This point is different from the conventional model \[11\] and will play the important role for precise torque estimation without core-loss resistance measurement.

3. Torque Estimation without Core-Loss Measurement and Position Encoder

3.1 Torque Estimation Using Active and Reactive Powers

This section proposes torque estimation strategy without core-loss measurement and position encoder, which enables us to reduce time-consuming for complicated measurement or identification of parameters. Torque estimation based on the power flow is generally known to be one of the straightforward approaches.

Namely, the active power is expressed by:

$$
P = v_1 d i_d + v_1 q i_q = v_1 d i_n + v_1 p i_p = \omega_r \lambda_{1d} i_d + R_i |i_d|^2 + R_m |i_d|^2 + \lambda_{1d} i_d = L_{1q}$$

and then, the electrically developed torque can be estimated as:

$$
\tau_e = \frac{\lambda_{1d} i_d}{P - R_i |i_d|^2 - R_m |i_d|^2 + \lambda_{1d} i_d / L_{1q}}$$

The above estimation strategy requires $R_m$, which extremely varies according to the amplitude and the phase in the stator current vector, and rotor speed. As a result, core-loss should be measured in all operating range, resulting in much time-consuming and costs. This motivates us to tackle the torque estimation without core-loss measurement.

Eq. (14) offers the reactive power $Q$ as follows:

$$
Q = v_1 d i_q - v_1 q i_d = v_1 d i_n - v_1 p i_p = \omega_r L_{1q} |i_d|^2 + \omega_r \lambda_{1d} i_d = -R_m \tau_e / L_{1q}$$

It should be noted that the reactive power depends on the equivalent core-loss resistance $R_m$ as well as the $q$-axis inductance $L_q$ since $R_m$ is the coefficient of $\lambda_{1d}$ in the case of the proposed extended-flux model as shown in Eq. (14). Substituting Eq. (17) into Eq. (16) to eliminate $R_m$ yields the following quadratic equation with respect to the developed torque $\tau_e$:

$$
\omega_r \tau_e \omega_r^2 + (P - R_i |i_d|^2) \tau_e + (\omega_r L_{1q} |i_d|^2 + \omega_r \lambda_{1d} i_d - Q(L_{1q} |i_d|^2 + \lambda_{1d} i_d))^2 = 0
$$

Fig. 2 shows the configuration of the proposed torque estimation algorithm. It should be noted that the above equation can give the developed torque without $R_m$ and $L_d$, which extremely varies with the operating point. Hence, the proposed method can realize robust torque estimation with respect to magnetic non-linearity and core-loss variation.

Eq. (18) requires, however, the rotor speed $\omega_r$ and the extended-flux $\lambda_{1d}$ as well as rotor position $\theta_{re}$ to reconstruct $i_d$ through the coordinate transformation. In this paper, these quantities are estimated by the adaptive full-order observer based on the extended-flux model without the equivalent core-loss resistance \[9\]. The following subsection describes the detail of this adaptive full-order observer.

3.2 Construction of Adaptive Full-Order Flux Observer for Torque Estimation without Position Encoder

To realize accurate and robust estimation of the extended-flux $\lambda_{1d}$ and rotor position $\theta_{re}$, the flux observer should be constructed while considering:

- Robust flux and rotor position estimation with respect to magnetic saturation effect;
Position Sensorless Torque Estimation of Synchronous Reluctance Motors
Based on Extended-Flux Model without Core-Loss Measurement

Accurate measurement of equivalent core-loss resistance.

Previously, a flux observer on the stationary coordinate (α-β axes) has already been proposed [4]. However, the observer in Ref. [4] consists of the voltage model using a pure integration and flux model expressed on d-q coordinate, in which \( L_d \) and \( L_q \) take much complicated non-linearity into account. In this paper, the previous section has reviewed that the extended-flux model is insensitive to magnetic non-linearity. This paper, hence, constructs the extended-flux model based observer for torque estimation [9] to succeed the robustness in this model with respect to magnetic saturation effects.

The equivalent core-loss resistance, however, is the unknown parameter in the advance of the construction of the extended-flux observer. Therefore, this observer can not be constructed based on the extended-flux model with equivalent core-loss resistance in Eq. (14). This paper tackles this serious problem with the assistance of the adaptive flux observer based on the extended-flux model without the equivalent core-loss resistance according to the notion in Ref. [12], which has pointed out for an induction motor that:

1. The flux observer with appropriate design can suppress influences of speed estimation error or rotor resistance mismatch;
2. Influences of the un-modeled core-loss resistance is equivalent to the disturbance caused by above mismatches.

This implies that the proposed full-order flux observer [13] can be less sensitive to influences of the un-modeled core-loss resistance. This subsection shows the adaptive full-order flux observer for SynRM, which can be described without \( L_d \) and \( R_m \).

Let \( v_i, i_j \) and \( \lambda_j \) be the voltage \( v_{i\alpha} + jv_{i\beta} \), the current \( i_{\alpha} + j_i \) and the flux \( \lambda_{\alpha} + j_\lambda \) on the α-β axis, respectively. Defining \( x = [i \lambda]^T \), the state equation of SynRMs on the α-β coordinate fixed to the stator is expressed by:

\[
px = Ax + Bv
\]

\[
i = C_1x
\]

in which:

\[
A = \begin{bmatrix} -\frac{R}{L_q} & \frac{j\omega_m}{L_d} \\ 0 & \frac{j\omega_m}{L_q} \end{bmatrix}
\]

\[
B = \begin{bmatrix} 1 \end{bmatrix}
\]

\[
C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}
\]

In this case, \( p|\lambda\) is assumed to be negligible since \( i_d \) can be converged on its reference immediately by the current controller, although the reference for this current varies according to the torque command.

Based on Eq. (19), the full-order flux observer to estimate flux \( \lambda \) is constructed as:

\[
p\dot{x} = A\dot{x} + Bv + H(i - i)
\]

\[
\dot{\lambda} = C_2\dot{x}
\]

in which \( \dot{x} \) stands for the estimated value of \( x \), and

\[
A = \begin{bmatrix} -\frac{R}{L_q} & \frac{j\omega_m}{L_d} \\ 0 & \frac{j\omega_m}{L_q} \end{bmatrix},
\]

\[
C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad \text{and}
\]

\[
H = \begin{bmatrix} H_1 & H_2 \end{bmatrix}^T
\]

represents the observer gain to stabilize state estimation. This design parameter dominates the stability and the sensitivity of parameter mismatch on flux estimation. In this paper, the observer gain \( H \) is designed so that the poles of the observer might be reassigned on the real axis of the left-half plane in the \( s \) domain to realize stable and robust flux estimation with respect to un-modeled coreloss resistance. Refer to Ref. [14] for details. In addition, this adaptive observer can identify the rotor speed by the following conventional adaptive scheme [13]:

\[
\dot{\omega}_s = \left( K_p + \frac{K_i}{s} \right) \left( e_1 \lambda \right)
\]

in which \( e_1 \) stands for estimation error in current.

Fig. 3 shows the configuration of the full-order flux observer. Again, it should be noted that \( L_d \) variation does not affect this flux estimation because this flux observer can be constructed without \( L_d \). As shown in Fig. 3, this observer utilizes voltage knowledge \( v \) as well as current \( i \) to estimate flux \( \lambda \), which enables to eliminate \( L_d \) from the standard model. Therefore, the proposed observer requires the voltage control accuracy of the PWM inverter or the voltage detectors of the PWM inverter output, as well as the current detectors.
4. Experiments

This section describes the experimental results on core-loss resistance estimation and discusses torque estimation performance for the test SynRM (3 Φ-200 V, 1 kW, 1,800 min⁻¹) in position sensorless control. The following experiments were carried out with a digital signal processor TMS320C6713B by TI, in which the motor speed and current amplitude were kept constant by a load machine and current regulator for test SynRM. Under this condition, torques of test motor were estimated by the proposed algorithm and measured by the torque sensor (Magtrol: TMB306/41) while intentionally adjusting current control phase β from 39 to 69 deg.

4.1 Estimation Results and Discussion of Equivalent Core-loss Resistance

Fig. 4 demonstrates estimation results of equivalent core-loss resistance under different speeds and current amplitudes. This approach can estimate equivalent core-loss resistance by the simultaneous solution of Eqs. (15) and (17) with respect to \( R_m \). Although true values of this resistance are unknown, these results would be feasible because:

- The larger the motor current flows, the larger \( R_m \) becomes;
- The higher the motor speeds up, the larger \( R_m \) becomes;
- The more the current control phase \( β \) advances, the smaller \( R_m \) becomes, since stator flux amplitude decreases gradually.

4.2 Estimation Results and Discussion of Developed Torque

Figs. 5-7 show torque estimation results, in which \( τ \) and \( \hat{τ} \) mean the measured torque and estimated values in mechanical, respectively, and the estimated values with the assistance of the position encoder are also illustrated by \( \hat{τ} \). This paper aims at the realization of MTPA (maximum torque per ampere) and precise estimation for the desirable current control phase as the first step.
Fig. 5  Torque estimation results at 450 min$^{-1}$.

Fig. 6  Torque estimation results at 900 min$^{-1}$.

Fig. 7  Torque estimation results at 1350 min$^{-1}$.

Table 1  Estimation performances at maximum torque under position sensorless control.

<table>
<thead>
<tr>
<th>$w_{rot}$ (min$^{-1}$)</th>
<th>$i_1$ (A)</th>
<th>$\tau_{\text{max}}$ (N·m)</th>
<th>$\tau_{\text{max}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>6</td>
<td>2.11($\theta = 51^\circ$)</td>
<td>1.99($\theta = 51^\circ$)</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3.91($\theta = 57^\circ$)</td>
<td>3.73($\theta = 54^\circ$)</td>
</tr>
<tr>
<td>900</td>
<td>6</td>
<td>2.10($\theta = 51^\circ$)</td>
<td>2.00($\theta = 51^\circ$)</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3.74($\theta = 57^\circ$)</td>
<td>3.72($\theta = 54^\circ$)</td>
</tr>
<tr>
<td>1,350</td>
<td>6</td>
<td>2.07($\theta = 51^\circ$)</td>
<td>1.99($\theta = 51^\circ$)</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3.68($\theta = 57^\circ$)</td>
<td>3.70($\theta = 54^\circ$)</td>
</tr>
</tbody>
</table>

It can be seen from these figures that the top of $\hat{\tau}$ is much consistent with that of $\tau$ although some extent of error in torque estimation is visible. Table 1 demonstrates the estimation performances for maximum developed torque and current phase for MTPA control. It turns out from this table that estimation error in current control phase $\beta$ is three degrees at most for all experiments, which does not seriously matter because the developed torque hardly dips by this estimation error in $\beta$.

These results conclude that the proposed torque estimation is much suitable strategy for a preparation of
look-up table for MTPA control.

5. Conclusions

This paper has proposed the extended-flux model considering the equivalent core-loss resistance and the developed torque estimation without $L_d$ and $R_m$ measurements, which makes it possible to reconstruct the torque characteristics robustly with respect to magnetic nonlinearity and core-loss. Finally, this paper has demonstrated the feasibility of the proposed strategy by some experimental results, which conclude that the proposed method is suitable for MTPA control of SynRMs.

References