Control of Z-Source Inverter Connected to a Single-Phase AC Utility System

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Received: June 28, 2010 / Accepted: September 13, 2010 / Published: May 31, 2011.

Abstract: This paper presents a single-phase Z-source inverter as a power conditioning system for a single phase utility connected system. Z-source inverter is a single-stage topology that has buck-boost feature, which is possible because of additional shoot through state introduced in zero state of the conventional inverter pulse width modulation and provides desired output AC voltage. Small distributed generation (DG) system with alternate energy sources requires power conditioning units with low cost, high efficiency and tolerance to wide range of input voltage variation and has to perform various functions such as dc-ac conversion, system control and achieve power quality norms. To meet some of these requirements a two-loop control strategy for ac side control with grid current feedback with PI control and inner filter capacitor current feedback with proportional control and on dc side PID control for Z-source capacitor voltage regulation are employed, which gives good transient response also suppress load and source disturbances effectively. Theoretical analysis of proposed scheme is established and then simulation results are presented to validate proposed control strategy.

Key words: Buck-boost, Z-source inverter, distributed generation, phase locked loop.

1. Introduction

The growth of an economy depends on energy availability. Due to depletion of fossil fuels and their environmental effects, with growing load the utilities are stressed and environmental concerns made to look into renewable energy sources such as Fuel cells, PV and Wind etc.. The electrical energy from renewable energy sources comes under distributed generation (DG) and is a small capacity power source at customer site at distribution voltage levels to serve part or all of customer load as backup power. This type of DG solutions finds applications at substations to reduce peak load demand and defer substation capacity reinforcement [1].

The heart of a DG system is the power conditioning unit, which has an inverter that converts DC to AC power fed into either utility or directly used to meet the local load demand, if it is a stand-alone system. Usually, the available power from fuel cells and photovoltaic cell is of DC in nature and their terminal voltage widely varies. Moreover, the power conditioning system should be of high efficiency, able to cope with wide input voltage variations and meet the required IEEE standards [2, 3] with reference to power quality.

Pulse-width modulated voltage source inverter (VSI) play a vital role in DC-AC power conversion, interconnection, control and power quality aspects. The current control of PWM inverters is advantageous over voltage control in terms of control of instantaneous current waveform, high accuracy, peak current protection, and good dynamic response [4] etc.. Hence, closed-loop regulation of PWM inverters is critical in the power conditioning systems such as in UPS, automatic voltage regulators (AVR), and programmable ac source (PACS).

Various topologies for power conditioning systems,
in of DG system, have been reported in literature such as single-stage and multi-stage inverters with or without galvanic isolation and buck-boost capability [2]. Conventional PWM inverters are buck type inverters and require additional power stage to boost the voltage from the renewable source. Hence, there is a need to develop buck-boost type of inverters for renewable energy based DG systems. In this context, simple topologies such as single stage inverters with fewer components, which can accept wide range of input voltages, and possess buck-boost capability, are receiving considerable attention in these days. One such configuration employing z-source network on the input side of conventional buck type inverter is proposed in Ref. [5]. It is capable of giving bucking as well as boosting operation in single-stage conversion. Hence, this fact reduces switching loss and increases the efficiency, which is very much required in case of DG applications.

This paper presents a two-loop control scheme for closed-loop regulation of single-phase Z-source inverter with LCL filter for a power conditioning system, based on maximum boost control. The section 2 gives introduction to Z-source inverter and modified modulation with respective to traditional VSI, section 3 describes control of Z-source inverter and section 4 discusses the results, conclusions are drawn in section 5.

2. Analysis of Z-Source Inverter

2.1 Topology of Z-Source Inverter

A Z-source inverter, shown in Fig. 1, has special feature or state called as shoot-through state, that is not present in conventional VSI, which allows both switches of same leg to be on simultaneously, and gives voltage boost capability to the inverter without damaging the switching devices. It employs an impedance network to couple the converter main circuit to the power source, load or another converter which gives additional features that are not inherently present in conventional voltage or current source converters. The presence of Z-source network comprising two inductors and two capacitors, allows shoot through of switches in same leg. Furthermore, it also provides buck-boost feature for wide range of input voltages [5].

Voltage boost capability of ZSI, shown in Fig. 1, is due to the energy transfer from capacitors to inductors, during the shoot through state. Since, the capacitors are charged to higher voltages than the source voltage, the diode ‘D’ prevents discharging of capacitors through the source.

2.2 Mathematical Analysis of Z-Source Inverter

Considering a symmetric Z-source network i.e. \( L_1 = L_2 = L \) and \( C_1 = C_2 = C \) (also \( V_{C1} = V_{C2} = V_C \) and \( V_{L1} = V_{L2} = V_L \)), the effect of introducing shoot through within the zero state of PWM switching cycle is analyzed by considering equivalent circuits, shown in Fig. 2, of Z-source during shoot through and non-shoot through states. The inverter side of Z-source network can be treated as an equivalent current source with a finite current during non-shoot through active state and a zero current during shoot through zero state. From the equivalent circuit Fig. 2(a), during non-shoot through state ‘\( T_{NS} \)’ inverter circuit is represented by current source and dc source is connected to ac load. Using circuit theory principles, various mathematical expressions relating various voltages can easily be obtained as:

\[
\begin{align*}
V_d &= V_{dc} \\
V_L &= V_{dc} - V_C \\
V_s &= V_C - V_L = 2V_C - V_{dc}
\end{align*}
\]

(1)

Fig. 1 Single phase Z-source inverter.
From the equivalent circuit Fig. 2(b), during shoot through state ‘TST’ inverter circuit is represented by short circuit, we have:

\[
\begin{align*}
V_C &= v_t \
V_d &= V_C + v_e = 2V_C
\end{align*}
\]

where ‘T’ is the half-period of the carrier wave, ‘TN’ and ‘TST’ are the non-shoot through and shoot through times respectively and \( T = T_{NS} + T_{ST} \). Applying volt-sec balance to the Z-source inductor over one switching cycle, the capacitor voltage expression of Z-source network can be obtained as:

\[
V_C = \left( \frac{T_{NS}}{T_{NS} - T_{ST}} \right) V_{dc} = \left( \frac{1 - \left( \frac{T_{ST}}{T} \right)}{1 - 2 \left( \frac{T_{ST}}{T} \right)} \right) V_{dc} \tag{3}
\]

Using Eqs. (1) and (3), the peak DC-link voltage across inverter bridge is obtained as:

\[
\Lambda V_i = 2V_C - V_{dc} = \left( \frac{1}{1 - 2 \left( \frac{T_{ST}}{T} \right)} \right) V_{dc} = BV_{dc} \tag{4}
\]

Peak phase voltage of ZSI and using Eq. (4):

\[
\Lambda V_{ac} = m(V_i / 2) = m \left( BV_{dc} / 2 \right) = B(mV_{dc} / 2) \tag{5}
\]

where ‘B’ is boost factor, \( \geq 1 \), due to shoot through state and ‘m’ is modulation index of conventional VSI. From Eqs. (3) and (4), the shoot through duty ratio, ‘d’ = \( T_{ST}/T \) has the operating range of \( 0 \leq d < 0.5 \), for which \( \hat{v}_i > 0 \).

### 2.3 Modulation of Z-Source Inverter

The buck-boost capability of ZSI is on account of shoot through duty ratio of appropriate magnitude, \( d = \left( T_{ST} / T \right) \), within zero state of the switching cycle. The function of PWM strategy is to get the active pulse width within each carrier interval that has same volt-sec average as the original reference over the same interval. But the total harmonic distortion (THD) of the output depends on the location of pulse within the carrier interval, which does not affect its cumulative volt-sec average over that interval [6]. It makes use of complete available operating time of a PWM inverter.

The PWM modulation of Z-source inverter is modified accordingly by meeting the above criteria i.e. the shoot through states are inserted within the zero states without affecting the volt-sec average and are added adjacent to the instants of state transitions as shown in Fig. 3, keeping No. of device commutations, and duration of active state unchanged per switching cycle [7]. Hence, zero state (includes shoot through state) time is not only used to control the average voltage, but also provides buck-boost feature to the inverter.

### 3. Two-Loop Control of Z-Source Inverter

The presented Z-source inverter is suitable for various applications such as motor drives, hybrid vehicles, and also in distributed generation. The overall system configuration for the utility connected application using ZSI with two interdependent control loops for Z-source capacitor voltage and inverter system is shown in Fig. 4. With ZSI, there is no need of an additional DC-DC converter.

In general, the controller is designed to track a pre-specified quantity, voltage or current to the set point reference, and regulation against disturbances. In case of Z-source inverter, shoot through duty ratio and modulation index are the two important control variables available to the designer by means of which one can use its buck-boost feature to realize the desired load voltage.
3.1 PID Controller Design for DC-link Voltage of Z-Source Inverter

In case of Z-source inverter the dc side controller is useful to provide constant average dc link voltage from a varying DC input voltage, which is case with DG sources, to the inverter. In order to regulate load voltage, the Z-source capacitor voltage needs to be controlled so that the effective DC-link voltage of the inverter maintained to a constant value. Hence, on DC side of the ZSI, the Z-source capacitor voltage is taken as a controlled quantity and the corresponding control scheme is shown in Fig. 5. In ZSI two different quantities need to be controlled, the first one is inverter load voltage, while the second one is the inverter DC-link voltage. Inverter load voltage is controlled by means of active duty ratio of switching devices, while the DC-link voltage is controlled by the shoot through duty ratio. From the equivalent circuits, shown in Fig. 2, it is clear that the DC-link voltage appears as a load voltage for the Z-source network. If there is any change in the DG source voltage, firstly the corresponding variation can be seen on the DC-link voltage, later on the inverter load voltage. In order to avoid the load voltage variations, the DC-link voltage needs to be regulated against changes in DG source voltage by controlling any one of the capacitor or inductor voltage of the Z-source network. Since the capacitor voltage is unidirectional, DC nature, and its measurement is much easier than the inductor voltage measurement. In view of this feature the Z-source capacitor voltage regulation loop is chosen here for maintaining the DC-link voltage. Since the capacitor charging depends on the shoot through duty ratio, there is need to establish a linear model, showing dependence on these quantities, for control loop design. In this context, the small-signal model of the ZSI is obtained in the form of signal-flow graph [8, 9] shown in Fig. 6, and it is

![Diagram of the Z-source network with control scheme](image-url)

Fig. 5 PID control scheme for Z-source capacitor voltage regulation.

![Signal flow graph model of Z-source network](image-url)

Fig. 6 Signal flow graph model of Z-source network.
developed under the following assumptions: (i) switching devices are ideal, (ii) inverter part of circuit as a current source (in Fig. 2), (iii) sum of shoot through duty ratio (d) and modulation index (m) equal to unity, (iv) average inverter current equal to Z-source inductor current i.e. average Z-source capacitor current is zero, (v) symmetric Z-source network i.e. \( L_1 = L_2 = L \) and \( C_1 = C_2 = C \), hence voltage across z-source capacitors and inductors are \( V_{c1} = V_{c2} = V_c \) and \( V_{l1} = V_{l2} = V_l \) respectively. The control to Z-source capacitor voltage transfer function Eq. (6) is obtained from the small-signal model and a digital PID controller [10, 11] is designed such that the loopgain transfer function gives a gain margin (GM) more than 6 dB and a phase margin (PM) between 30° and 60°.

The control to Z-source capacitor -1 voltage transfer function is given by

\[
G_{vc}(s) = \frac{\frac{V_c(s)}{V_{c1}(s)}}{\frac{V_c(s)}{V_{c1}(s)}} = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_3 s^3 + b_2 s^2 + b_1 s + b_0} (6)
\]

where the coefficients are given as

- \( a_3 = -G_L L_2 C_1 \)
- \( a_2 = C_2 L_2 G_L \)
- \( a_1 = C_2 L_2 G_L G_D \)
- \( a_0 = -(1-2d)G_L L_2 G_D + G_D L_2 \)
- \( b_3 = C_1 C_2 G_D \)
- \( b_2 = -(1-2d)C_1 C_2 \)
- \( b_1 = -(1-2d)C_1 C_2 G_D \)
- \( b_0 = -(1-2d)^2 \)

By using assumptions (iii), (iv) and (v), Eq. (5) is obtained in simplified form as:

\[
G_{vc}(s) = \frac{\frac{V_c(s)}{V_{c1}(s)}}{\frac{V_c(s)}{V_{c1}(s)}} = \frac{a_3 s^3 + a_2 s^2 + a_1 s + a_0}{b_3 s^3 + b_2 s^2 + b_1 s + b_0} (6)
\]

The control to capacitor voltage transfer function has one RHP zero \((z = 2.53 \times 10^4)\) and a complex conjugate pole-pair \((p = -461 \pm j1470)\). In view of this, its frequency response magnitude plot is flat up to the corner frequency of complex pole-pair and then at high frequencies, it is drooping at -40 dB/decade. On account of RHP zero the phase plot is going towards -270°. The simple controllers such as one pole and one zero configurations may not provide enough flexibility for the control loop design and in some cases it may not possible to realize relative stability determining gain and phase margin requirements simultaneously. Although, many higher order compensators can easily be designed, but their implementation increases the computational burden on the processor.

A two pole two zero compensator, PID type controller, is designed in this paper that is simple to realize and yet easy to meet the stability margins requirement. A pole at origin gives the drooping nature to the loop-gain in the low frequency region and eliminates steady-state error in the regulation problem. A zero (at 15.2 Hz) and then pole (at 24 Hz) is placed in the frequency region to reshape the loop-gain transfer function. Controller pole-zero locations are adjusted using SISOTOOL [12], and tuned to achieve GM of 18.3 dB and PM of 46.3° for the loop-gain. The resulting PID controller in s-domain is

\[
G_c(s) = \frac{0.0727(s + 95.5)(s + 420.97)}{s(s + 150.53)} (8)
\]

The above s-domain transfer function is now transformed into discrete domain using Tustin transformation, a digital redesign approach, and the final digital PID controller for the control of Z-source capacitor voltage for the parameters listed in Table 1, is obtained as Eq. (9). The frequency response plot of control to Z-source capacitor voltage transfer function, controller and loop-gain transfer functions are shown in Fig. 7.

\[
G_c(z) = \frac{0.07358(z - 0.9937)(z - 0.9723)}{(z - 1)(z - 0.99)} (9)
\]

The above designed dc side PID controller regulates the Z-source capacitor voltage, input to the inverter, and provides good dynamic response against disturbances.

### 3.2 Control of AC-Side of Z-Source Inverter

The AC side control employs current control strategy, which has two important aspects, one is reference current generation to produce PWM signals and current controlling to generate modulation index.
Table 1 Parameters of ZSI system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC source voltage, (V_{dc})</td>
<td>50 V</td>
</tr>
<tr>
<td>Z-source inductor, (L_1 = L_2 = L)</td>
<td>r = 0.15 Ω; L = 380 μH</td>
</tr>
<tr>
<td>Z-source capacitor, (C_1 = C_2 = C)</td>
<td>r_c = 0.2 Ω; C = 400 μF</td>
</tr>
<tr>
<td>Shoot through duty ratio, (D)</td>
<td>~0.2</td>
</tr>
<tr>
<td>Filter inductance (Inverter side)</td>
<td>r = 0.125 Ω; L_i = 460 μH</td>
</tr>
<tr>
<td>Filter inductance (Utility side)</td>
<td>r = 0.1 Ω; L_g = 115 μH</td>
</tr>
<tr>
<td>Filter capacitor, (C_f)</td>
<td>2.5 μF</td>
</tr>
<tr>
<td>Utility voltage, (V_g)</td>
<td>V_g = 38 V</td>
</tr>
<tr>
<td>AC supply/ Switching frequency</td>
<td>50 Hz/15 kHz</td>
</tr>
</tbody>
</table>

Fig. 7 Frequency response bode plots of G_{vcd}(s), G_c(s) and Loop-gain, T_c(s).

The AC current control of inverter is implemented using utility current feedback with PI control as outer current loop, and filter capacitor current feedback with a proportional controller in inner current loop [13]. The block diagram of the grid current control scheme is shown in Fig. 8. The outer feedback loop i.e. grid current feedback with PI control generates reference to the inner loop for filter capacitor current feedback, which then decides inverter output voltage, along with insertion of shoot through duty ratio derived from PID controller of Z-source capacitor voltage in DC-link.

Here, the current reference is being generated by using phase locked loop (PLL). The PLL is feedback control system that adjusts the phase, locally generated ac signal to match the phase of an input signal. Further, the PLL also provides unity power factor operation which involves synchronization of inverter output current with grid voltage and gives a sinusoidal current reference.

To provide effective harmonic compensation, an LCL filter is employed as shown in Fig. 4. The LCL filter has certain advantages over the simple L-filter in terms of net size of inductance, cost and performance. The LCL filter, being a third order filter, suppresses higher order harmonics more effectively than a simple L-filter (first order filter). Assuming negligible parasitic components, the inverter voltage to output current transfer function with LCL-filter is given by

$$i_v(s) = \frac{1}{sL_iL_gC_f(s^2 + \omega_r^2)}$$

where \(\omega_r = \sqrt{\frac{L_i + L_g}{C_fL_iL_g}}\) is resonant frequency.

Fig. 9 shows frequency response obtained using MATLAB [12] of LCL and L-filters with same inductance, the higher order harmonics in case of LCL-filter are suppressed at a rate of -60 dB/decade, whereas -20 dB/decade for L-filter after the resonant frequency.

4. Discussion of Simulation and Experimental Results

The simulation for the proposed utility connected system, using Z-source inverter, is carried out using PSIM software [14]. The effectiveness of the control scheme is studied by creating source and load disturbances of utility connected Z-source inverter system.

4.1 Source Disturbance

Firstly, the PID controller effectiveness for Z-source capacitor voltage regulation is tested against source disturbance and capacitor voltage reference change.
Fig. 10 shows simulation results when input DC voltage is changed from 50 to 40 V (20% change). The original ZSI system is designed to operate at a shoot through duty ratio of ‘0.2’. Against the input disturbance, the shoot through duty ratio is changed from ‘0.2’ to ‘0.27’ in order to regulate the average DC link voltage. It can also be noted that the Z-source capacitor voltage is maintained constant.

Fig. 11 shows when input DC voltage is changed from 50 to 60 V (20% change). Now, the operating point of shoot through duty ratio is changed from ‘0.2’ to ‘0.08’ to regulate the output and correspondingly the Z-source capacitor voltage is also regulated. The utility current THD is found to be 2.5% in both cases, which satisfies the IEEE standards. The Z-source capacitor voltage reference is increased by 20% and the PID controller effectiveness is plotted for reference tracking as shown in Fig. 12.

4.2 Load Disturbance

The local R-load connected to the system is perturbed from 50 → 25 Ω at 0.125 sec is shown in Fig. 13. From the figure, the shoot through duty ratio and Z-source capacitor voltage are unchanged, this implies that the load disturbances are not transferred to the dc side and these variations are regulated by adopted control scheme. The utility current THD is found to be 2.4%.

4.3 Experimental Results

To verify the boosting concept, prototype ZSI has been constructed and then experimented for the sinusoidal modulation index, m = 0.7 and shoot through duty ratio of d = 0.2 in open-loop condition. It can be seen that, from Figs. 14 and 15, the load voltage is sinusoidal with THD of 1.8% that falls within the acceptable range as per IEEE standards.
5. Conclusions

The application and control of Z-source inverter for a grid connected system has been presented in this paper. A digital PID compensator has been designed to regulate the Z-source capacitor voltage that shows good transient response and reference tracking for 20% input variation and a multi-loop controller is designed for entire system with utility current feedback with PI control and a proportional controller for inner filter capacitor current feedback on a AC side, to get good dynamic response. Simulation results show good reference tracking and disturbance rejection for input variations. Experimental results were given for open-loop operation of the ZSI indicating lower THD and load voltage is almost sinusoidal in nature.

References