MPAM Signal Processing by Generalized Receiver under Nakagami Fading

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Abstract: We derive the closed-form expressions for the bit error rate (BER) performance of M-ary pulse amplitude modulated (MPAM) signal constellations under implementation of the generalized detector (GR), which is constructed based on the generalized approach to signal processing in noise, as a function of the analog-to-digital converter (ADC) word length, the signal-to-noise ratio (SNR), and the fading distribution. These results allow us rapidly and accurately to evaluate the system performance when the ADC resolution is limited, as is generally the case in high sampling rate wireless communication systems, and thus provide a useful tool for wireless communication system design based on the generalized approach to signal processing in noise, analysis and optimization.

Key words: Generalized receiver, additive white gaussian noise, nakagami fading, bit error rate, pulse amplitude modulation.

1. Introduction

Direct conversion transceivers employing single-carrier (SC) schemes with directional antennas to counter intersymbol interference (ISI) are a promising alternative to multicarrier (MC) schemes, for high data transmission in wireless communication systems operating in the 60 GHz and 100 GHz regime and employing the generalized receiver (GR) constructed based on the generalized approach to signal processing (GASP) in noise [1-6]. In-phase and quadrature-phase (I/Q) imbalance is one of the major problems associated with direct conversion transceivers [7], which can be alleviated to an extent by employing one dimensional signal constellations [8].

It is well known that the power consumption in an analog-to-digital converter (ADC) is directly proportional to $f_s \times 2^N$, where $f_s$ is the sampling frequency and $N$ represents the ADC word length. Thus, with sampling rates in the range of Giga samples per second, $N$ must be kept as small as possible. Monte Carlo simulations are often used to evaluate the system performance in terms of average probability of error in the presence of limited ADC resolution.

In Ref. [9] an analytical expression for the bit error rate (BER) of orthogonal frequency division multiplexing (OFDM) based on MC schemes is presented, which is based on the assumption that ADC quantization noise is Gaussian distributed. This assumption is valid only when the OFDM wireless communication system employs a large number of sub-carriers and in general is not true for SC schemes.

In this paper, we derive closed form expressions for the BER at the GR output in the case of Gray coded Mary pulse amplitude modulated (MPAM) signal constellations for SC transmission in the presence of ADC quantization noise over the additive white Gaussian noise (AWGN) and fading channels with Nakagami distribution in wireless communication systems. With the novel derived expressions, 2PAM, i.e., binary phase shift keying (BPSK) and 4PAM are used as examples for the performance evaluation in...
various fading cases. For fading analysis, the Nakagami distribution is assumed which covers line-of-sight channels. To the best of our knowledge, the closed form expressions for MPAM schemes in the presence of ADC quantization noise, AWGN and fading have been reported in Ref. [10] only and we use the same approach to analyze the GR BER employed by wireless communication system for the considered cases.

2. System Model

The real baseband equivalent received signal $y$ of an MPAM wireless communication system employing the GR, after coherent detection, operating under the influence of fading $h$, AWGN $n$, and in the presence of ADC quantization noise $z$ is given in the following vector form

$$y = hs + n + z = hs + \zeta$$

where $s$ denotes transmitted symbol vector chosen from the signal set

$$S = \{s_0, ..., s_{M-1}\}$$

The AWGN is assumed to be zero mean with the finite variance $\sigma_n^2$ and one-sided power spectral density $0.5N_0$. The additive quantization noise $z$ is uniformly distributed within the following limits [11], i.e.

$$z = \{z_1, ..., z_{M-1}\} \in \left[ -\frac{V}{2N}, \frac{V}{2N} \right]$$

and its variance is related with $N$ as follows

$$\sigma_z^2 = \frac{V^2}{3 \times 2^2 N}$$

where the ADC is assumed to be operating over a voltage range $[-V, V]$ volts. It is assumed that prior to ADC, an automatic gain control circuit brings the signal within $[-V, V]$ and there is no clipping.

In this paper, all signal constellations are assumed to be Gray coded and equally distributed within the limits of the interval $[-V, V]$ with an inter symbol distance of

$$d = \frac{2V}{M - 1}$$

Thus, the $l$-th symbol in the signal constellation set is given in the following form

$$s_l = \frac{V(2l - M + 1)}{M - 1}$$

The average constellation energy can be presented in the following form

$$E_s = \frac{1}{M} \sum_{l=0}^{M-1} s_l^2 = \frac{S_M V^2}{M(M - 1)^2}$$

where

$$S_M = \sum_{l=0}^{M-1} (2l - M + 1)^2$$

Thus we get

$$V = \sqrt{\frac{M E_s (M - 1)}{S_M}}$$

We assume a wireless communication system employing the GR with a flat fading. The independent and identically distributed fading amplitudes $h$ are drawn from a distribution with the probability density function (pdf) $f(h)$. Assuming perfect channel state information at the GR, symbol wise detection is performed, with the most likely transmitted symbol being chosen as the symbol, which minimizes the metric

$$C(y, s_k) = |y - s_k|^2$$

over all $s_k \in S$.

3. GR BER Performance Analysis

The SNR per symbol at the GR output is defined in the following form [2]

$$q = \frac{E_s}{2N_0}$$

The average energy bit $E_b$ is related with $E_s$ as

$$E_s = \log_2(M) \times E_b$$

The SNR per bit at the GR output thus can be presented in the following form

$$q_b = \frac{q}{\log_2(M)} = \frac{E_b}{2N_0}$$

In the fading case, the instantaneous and average SNRs are given as $h^2 q$ and $E[h^2] \times q$, where $E[\cdot]$ denotes the mathematical expectation operation. In this paper, we assume that
\[ E[h^2] = 1 \]  

### 3.1 AWGN and ADC Quantization Noise

Taking into account the results discussed in Ref. [12], we can define the GR output BER for Gray coded MPAM signal constellations in the presence of AWGN, which can be generalized for any additive noise \( \zeta \) in the following form:

\[
P_b = 2K_M \sum_{k=1}^{\log_2(M)} \sum_{i=0}^{\infty} \mathcal{X}(k,i,M) \left\{ \int \frac{f_{\zeta_2-\zeta_1}(x)dx}{\pi} \right\} \]

where \( f_{\zeta_2-\zeta_1}(x) \) is the background noise at the GR output [2] and the bit dependent function \( \mathcal{X}(k,i,M) \) is given in Eq. (24).

For better understanding Eq. (15), there is a need to recall the main statements of the GAS [1-6], based on which the GR is constructed. There are two linear systems at the GR front end that can be presented as bandpass filters, namely, the preliminary filter (PF) with the impulse response \( h_{PF}(\tau) \) and the additional filter (AF) with the impulse response \( h_{AF}(\tau) \). For simplicity of analysis, we consider that these filters have the same amplitude-frequency responses and bandwidths by value. Moreover, a resonant frequency of the AF is detuned relative to a resonant frequency of PF on such a value that signal cannot pass through the AF. Thus, the signal and noise can be appeared at the PF output and the only noise is appeared at the AF output.

It is well known fact that if a value of detuning between the AF and PF resonant frequencies is more than \( 4 + 5\Delta_{F_0} \), where \( \Delta_{F_0} \) is the signal bandwidth, the processes forming at the AF and PF outputs can be considered as independent and uncorrelated processes. In practice, the coefficient of correlation is not more than 0.05, but, in the case of signal absence in the input process, the statistical parameters at the AF and PF outputs will be the same, because the same noise is coming in at the AF and PF inputs and we may think that the AF and PF do not change the statistical parameters of input process, since they are the linear GR front end systems. By this reason, the AF can be considered as a generator of reference sample with a priori information a “no” signal is obtained in the additional reference noise forming at the AF output.

There is a need to make some comments regarding the noise forming at the PF and AF outputs. If the Gaussian noise \( n(t) \) comes in at the AF and PF inputs of the GR linear system front end, the noise forming at the AF and PF outputs is Gaussian, too, because AF and PF are the linear systems and, in a general case, take the following form

\[
\zeta_1(t) = n_{PF}(t) = \int h_{PF}(\tau)n(t-\tau)d\tau
\]

and

\[
\zeta_2(t) = n_{AF}(t) = \int h_{AF}(\tau)n(t-\tau)d\tau
\]

If, for example, AWGN with zero mean and one sided power spectral density \( \frac{N_0}{2} \) is coming in at the AF and PF inputs, the GR linear system front end, then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by Ref. [2]

\[
\sigma_n^2 = \frac{N_0\omega_0^2}{8\Delta_F}
\]

where, in the case if the AF (or PF) is the RLC oscillatory circuit, then the AF (or PF) bandwidth \( \Delta_F \) and resonance frequency \( \omega_0 \) are defined in the following manner

\[
\Delta_F = \pi f_L, \text{ } \omega_0 = \frac{1}{\sqrt{LC}}, \text{ } \beta = \frac{R}{2L}
\]

Based on these statements, we can write the pdf of the background noise at the GR output in the following form [2]

\[
f_{M=1}(x) = \frac{1}{2\sigma_n^2} K_0 \left( \frac{x}{2\sigma_n^2} \right)
\]

and

\[
f_{M=0}(x) = \frac{1}{2\sigma_n^2} \exp \left( -\frac{\mu x}{4\sigma_n^2} \right)
\]

where \( K_0(x) \) is the modified second kind Bessel function of an imaginary argument or, as it is also called, McDonald’s function, and \( \mu \) is the factor taking into consideration the parameters of PF and AF given by Eq. (19).
Now, return to Eq. (15). Here

\[ K_M = \frac{1}{M \log_2(M)} \]  

(22)

The modulation and bit dependent functions defined in Ref. [2] are seen to follow a regular pattern for Gray coded MPAM signal constellations and are given as

\[ \mathcal{K}(k, M) = (1 - 2^{-k}) M - 1 \]  

(23)

\[ \mathcal{K}(k, i, M) = (-1)^{i} \left[ \frac{2^{k-1}}{M} \right] \left[ \frac{1}{2} \right] \]  

(24)

\[ \mathcal{Y}(i) = 2i + 1 \]  

(25)

The combined pdf at the GR receiver back end of a sum of normally \( \mathcal{N}(0, \sigma_b^2) \) and uniformly \( U(0, \sigma_b^2) \) distributed random variables

\[ \zeta = n + z \]  

(26)

coming in at the GR receiver front end taking into consideration results in Ref. [13] can be defined in the following form

\[ f(\zeta) = \frac{2^{N-2}}{V} \left[ \text{erf} \left( \frac{\zeta + V \times 2^{-N}}{N_0} \right) - \text{erf} \left( \frac{\zeta - V \times 2^{-N}}{N_0} \right) \right] \]  

(27)

where

\[ \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt \]  

(28)

is the error function related to the Q-function as

\[ \text{erf}(x) = 1 - 2Q(\sqrt{2}s) \]  

(29)

Substituting (27) in (15) and solving the integral yields the expression of the BER for AWGN channels in the presence of ADC quantization noise at the GR receiver back end

\[ P_b = K_M \sum_{k=1}^{\log_2(M)} \sum_{i=0}^{\mathcal{K}(k, M)} \mathcal{K}(k, i, M) \times Q \left[ \frac{ME_b}{2N_0 \log_2(M)} \right] \times \left[ \frac{2 \times \mathcal{Y}(i) + (M - 1) \times 2^{-N+1}}{\sqrt{2M}} \right] \times \left[ 1 + \frac{\mathcal{Y}(i) \times 2^{N}}{M - 1} \right] \]  

\[ + K_M \sum_{k=1}^{\log_2(M)} \sum_{i=0}^{\mathcal{K}(k, M)} \mathcal{K}(k, i, M) \times Q \left[ \frac{ME_b}{2N_0 \log_2(M)} \right] \times \left[ \frac{2 \times \mathcal{Y}(i) - (M - 1) \times 2^{-N+1}}{\sqrt{2M}} \right] \times \left[ 1 - \frac{\mathcal{Y}(i) \times 2^{N}}{M - 1} \right] \]  

\[ - K_M \sum_{k=1}^{\log_2(M)} \sum_{i=0}^{\mathcal{K}(k, M)} \mathcal{K}(k, i, M) \times \frac{\sqrt{S_M}}{2^{N+1} \times (M - 1)} \frac{ME_b}{2N_0 \log_2(M)} \]  

\[ \times \left\{ \exp \left[ -\frac{ME_b}{2S_M N_0} \log_2(M) \times \frac{\mathcal{Y}(i) + (M - 1) \times 2^{-N}}{2} \right] \right\} \]  

\[ - \exp \left[ -\frac{ME_b}{2S_M N_0} \log_2(M) \times \frac{\mathcal{Y}(i) - (M - 1) \times 2^{-N}}{2} \right] \} \]  

(30)

As an illustrative example the BER of 2PAM (BPSK), by substituting \( M = 2 \) in (30) can be written in the closed form by the following way

\[ P_b = (0.5 + 2^{N-1}) Q \left[ \frac{E_b}{N_0} \log_2(M) \times (1 + 2^{-N}) \right] \]  

\[ + (0.5 - 2^{N-1}) Q \left[ \frac{E_b}{N_0} \log_2(M) \times (1 - 2^{-N}) \right] \]  

\[ - 2^{N-2} \frac{2N_0}{\pi E_b} \times \frac{1}{\log_2(M)} \]  

\[ \times \left\{ \exp \left[ -\frac{E_b}{2N_0} \times \log_2(M) \times (1 + 2^{-N})^2 \right] \right\} \]  

\[ - \exp \left[ -\frac{E_b}{2N_0} \times \log_2(M) \times (1 - 2^{-N})^2 \right] \} \]  

(31)

For the case when there is no additive quantization noise, i.e., \( N \rightarrow \infty \), (31) reduces to

\[ P_b = Q \left( \frac{E_b}{N_0} \times \log_2(M) \right) \]  

(32)

that is the exact expression for the error probability of BPSK in AWGN channels of wireless communication system employing the GR.

### 3.2 Fading, AWGN and ADC Quantization Noise

For the case of fading channels and coherent detection the signal constellation is scaled by a factor of \( h \). This implies that for a given fading realization \( h \), the Euclidian inter symbol distance is given as \( hd \).

Therefore, the BER evaluation requires the computation of an average over the fading distribution \( f(h) \) as

\[ P_b = \int_0^\infty P_b(h) f(h) dh, \]  

(33)

where \( P_b(h) \) denotes the instantaneous BER for a given fading realization \( h \) and is given as

\[ P_b = 2K_M \sum_{k=1}^{\log_2(M)} \sum_{i=0}^{\mathcal{K}(k, M)} \mathcal{K}(k, i, M) \int_0^\infty f(\zeta) d\zeta \]  

(34)
Following the derivation in the last section, it can be easily shown that, in principle, \( P_b(h) \) can be defined in the case of (26) in the following form:

\[
P_b(h) = K_M^{\log_{\Gamma(M)}\!\!(\xi(k, M))} \sum_{k=1}^{\log_{\Gamma(M)}\!\!(\xi(k, M))} \frac{\log_{\Gamma(M)}\!\!(\xi(k, i, M))}{2N_0} \left( \begin{array}{c} M_{E_0} \log_2(M) \\ \sqrt{2s_M} \end{array} \right) \times \left[ 2x + h \log_{\Gamma(M)}\!\!(\xi(k, (M-1)x) \times 2^{-N+1}) \right] \left[ 1 - \frac{h \log_{\Gamma(M)}\!\!(\xi(k, i, M)) \times 2^N}{M-1} \right]
\]

where \( \log_{\Gamma(M)}\!\!(\xi(k, i, M)) \times Q \left( \frac{ME_0 \log_2(M)}{2N_0} \right) \times \left[ 2x + h \log_{\Gamma(M)}\!\!(\xi(k, (M-1)x) \times 2^{-N+1}) \right] \left[ 1 - \frac{h \log_{\Gamma(M)}\!\!(\xi(k, i, M)) \times 2^N}{M-1} \right]
\]

By substituting

\[
h = \frac{1 + x}{1 - x}
\]

in (33), multiplying with

\[
\frac{\sqrt{1-x^2}}{1-x^2}
\]

and using the Gauss-Chebyshev quadrature [14], the BER over fading channels in the presence of AWGN and ADC quantization noise in wireless communications employing the GR can be approximated in the following form:

\[
P_b \approx \frac{2\pi}{L} \sum_{l=1}^{L} \frac{1 + x_l}{1 - x_l} \left( \frac{1 + x_l}{1 - x_l} \right) \sqrt{1 - x_l^2} \left( \frac{1 - x_l^2}{1 - x_l^2} \right) \]

where

\[
x_l = \cos \left( \frac{(2l-1)\pi}{2L} \right)
\]

and \( L \) denotes the degree of the Gauss-Chebyshev polynomial.

The large the value of \( L \) the closer is the approximation to the actual value. For our analysis, a value of \( L = 16 \) was found to be sufficient. The BER expression given in (38) is in general true for any fading distribution but we restrict our discussion to the Nakagami-\( m \) distribution. The pdf of the Nakagami-\( m \) distribution is given by

\[
f(h) = \frac{2m^m h^{2m-1}}{\Gamma(m)} \exp(-mh^2)
\]

where \( \Gamma(\cdot) \) denotes the Gamma function. It is good to note here that the Rayleigh distribution is a special case of the Nakagami-\( m \) (\( m = 1 \)) distribution.

### 4. Simulation Results

A comparison of the simulation and analytical results of 4PAM modulation for various \( N \) values and Rayleigh channels in wireless communication system employing the GR obtained by evaluating (38) is presented in Fig. 1. The closed form expression (38) is seen to be in good agreement with the Monte-Carlo simulation results. Equation (38) is general and can be used for the evaluation of the probability of error of any Gray coded MPAM signal constellation set. Comparative analysis of BER performance with wireless communication system in the case of implementation of the conventional receiver discussed in Ref. [10] is also presented in Fig. 1. A great superiority of GR employment in wireless communication system is obvious.

As another interesting application of the obtained results, the impact on BER by changing the ADC resolution of 2PAM for different \( m \) parameters at

\[
\frac{E_b}{N_0} = 10 \text{ dB}
\]

is shown in Fig. 2. This presentation can help the engineer of wireless communication systems implementing the GR to establish quickly the degradation in system performance due to limited ADC resolution for a given channel (\( m \) parameter). For example, a maximum BER of \( 10^{-4} \) dB can be achieved with 2-bit ADC at \( m = 8 \), while for a channel with \( m = 6 \) a four bit ADC would be required. Comparative analysis with the conventional receiver [10] employed by wireless communication system demonstrates a great superiority in favor of GR employment in wireless communication systems.
modulation schemes in the presence of ADC quantization noise has been analyzed for fading and non-fading scenarios for wireless communication systems employing the GR. Closed form expressions for the BER of Gray coded MPAM signal constellations in the presence of AWGN, Nakagami fading, and ADC quantization noise are presented. These expressions are then used to investigate the wireless communication system performance under GR implementation. This analysis enables the wireless communication system engineer to rapidly determine, for example, the minimum number of bits required to achieve a desired BER performance, in particular, a fading scenario. Comparative analysis with implementation of the conventional receiver [10] in wireless communication systems demonstrates us a great superiority of employment of the GR in wireless communication systems.

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